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SIXTH MONTHLY PROGRESS REPORT

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Effort during the reporting period, November 1 - 30, 1963, includes assembling the various subroutines into one complete program, preparing the program to run the test case supplied by Ames, and programming the classical turbulent boundary layer solution.

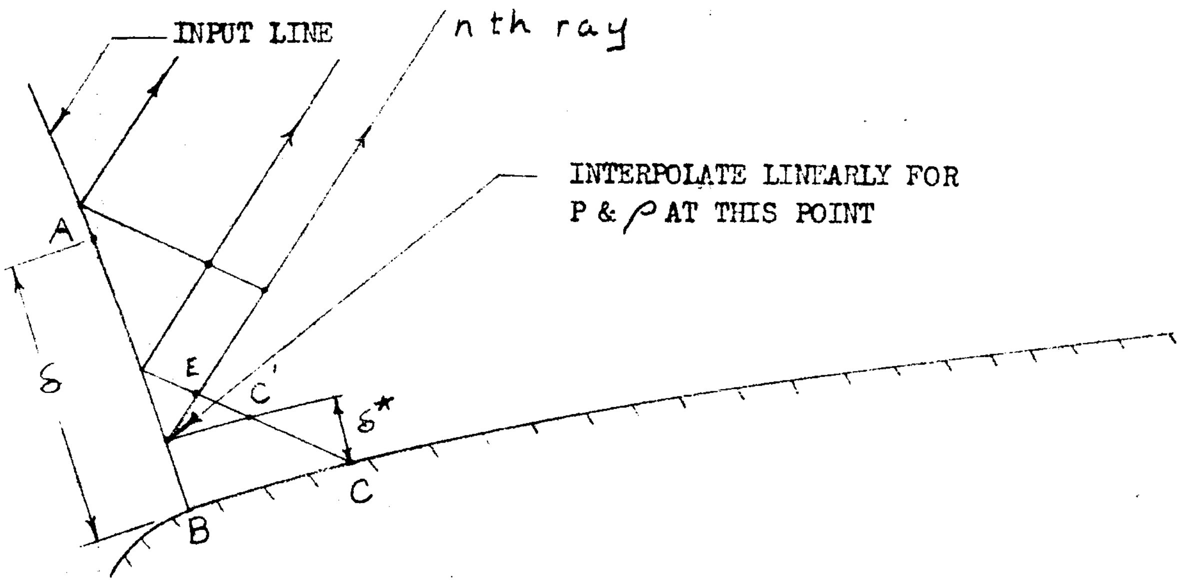
An attempt has been made to run the inviscid solution for the Mach 10.5 test case supplied by Ames. Some difficulty was encountered, however, due to intersection of the characteristic rays originating approximately 18 - 20 inches downstream from the nose of the ramp. These intersections are due to the small angles of the characteristic rays at these high Mach numbers plus the concave curvature of the ramp. Consequently, a special routine will have to be developed for handling possible coalescence of characteristic rays.

All of the various subroutines discussed in previous progress reports are being integrated into one complete program. This complete program is presently being checked out. A considerable amount of effort was involved in integrating the boundary layer solution with the characteristic solution. Some of the details of this integration are given in the following section.

INTEGRATION OF THE BOUNDARY LAYER AND CHARACTERISTIC SOLUTIONS

The boundary layer is computed in the stagnation region as discussed in the fifth progress report. Then from this solution the boundary layer characteristics are known at the input line. From this input line the boundary layer solution is integrated with the characteristic solution by the following procedure.

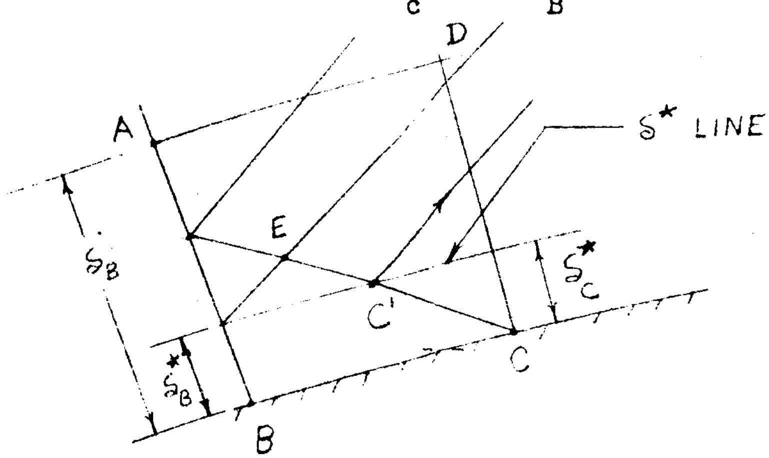
1. Compute characteristic rays from the input line to the shock and stop when the boundary layer displacement thickness δ^* is reached. This is called the nth ray. See fig. below.



2. Compute the body point C using the characteristics body point routine.

3. Curve fit the inviscid region bounded by the input line, bow shock and nth ray, plus the point C.

4. Now the actual body is displaced by the boundary layer displacement thickness δ^*. Starting at the input line $\delta^* = \delta_B^*$. Then it is assumed that $\delta_c^* = \delta^*_B$ at station C. See fig. below.



5. Locate the point D by assuming $\delta_c = \delta_B$.

6. The inviscid flow properties P_e and ρ_e at point D are obtained from the curve fit, step 3. The remaining flow properties required for the boundary layer solution are obtained from R gas. The velocity gradient is computed for the boundary layer.

$$\frac{\Delta U_e}{\Delta X} = \frac{U_D - U_A}{X_C - X_B}$$

X is measured along the surface.

7. Compute the boundary layer solution at station C using the flow properties from step 6. Then the boundary layer thickness δ and displacement thickness δ^* at station C are known. A new point D is known.

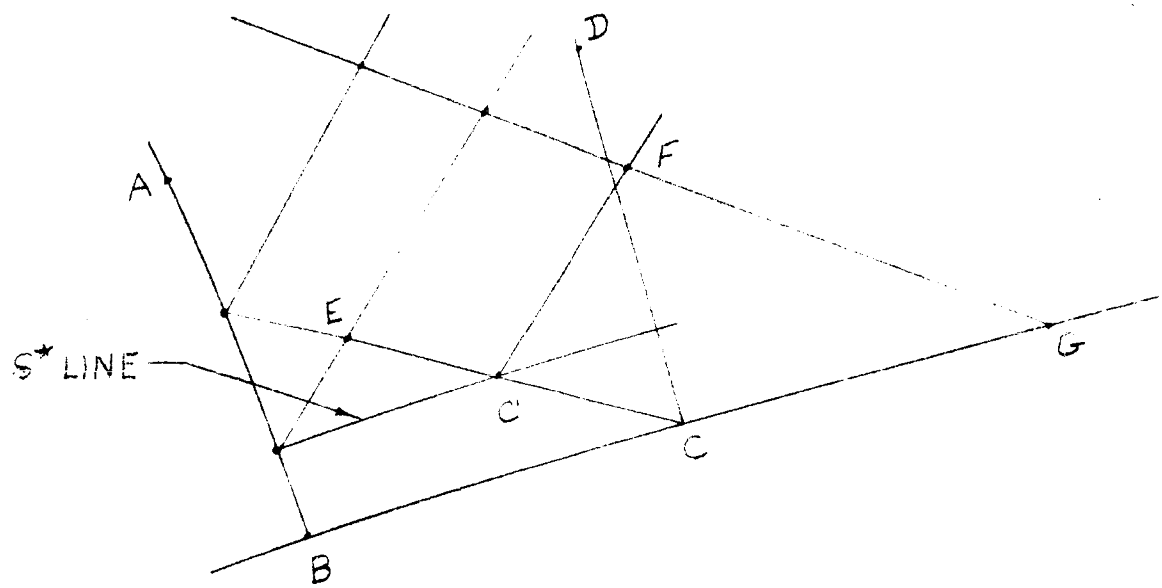
8. The inviscid flow properties P_e and ρ_e are again determined at point D of step 7 using the curve fit, step 3.

9. Steps 7 and 8 are repeated until the error in displacement thickness $\delta^* \leq 0.01$.

10. Now compute the new body point C' using the boundary layer displacement thickness from step 7. This body point C' is determined from the intersection of the ray $\overline{CE}$ and a straight line connecting the displacement thickness $\delta_B^* - \delta_C^*$.

11. The body flow direction for the characteristic solution is along $\delta_B^* + \delta_C^*$.

12. Compute the first family ray between the new body point C' and the shock wave, see fig. on following page.



13. Compute the new body point G starting at F in the above figure.
14. Repeat steps 4 - 14 for the new body point. Continue downstream until the shock corner or shock reflection is reached. Then transfer to the corresponding special routines. Each curve fit should include inviscid properties upstream of the first family ray passing through the previous C' point plus the newly computed inviscid body point. Such as G in above figure.

CLASSICAL TURBULENT BOUNDARY LAYER PROGRAM

This section describes the IBM computer program of the classical turbulent boundary layer method. This method treats a turbulent compressible boundary layer with pressure gradient, heat transfer, and dissociation and enables heat transfer, skin friction, boundary layer thickness, displacement thickness and momentum thickness to be computed for either thermodynamic equilibrium or frozen flow given the proper initial and boundary conditions.

As mentioned in the fifth progress report, it is convenient to treat a turbulent boundary layer by the integral method. This is due to the elimination of the turbulent shear stresses in this method. The following assumptions are made to make the problem amenable to an analytical solution.

1. All gas species considered behave as perfect gas species and the gas is assumed to be a binary mixture of atoms and molecules.
2. Radiation effects between the body and gas are negligible.
3. The flow is two-dimensional or axially symmetric.
4. The flow is in a steady state.
5. Prandtl's boundary layer equations are applicable.
6. The flow is assumed to be either in thermodynamic equilibrium or frozen state.
7. Thermodiffusion is negligible.

The momentum and energy boundary layer equations for a real gas, restricted to the above assumptions, reduce to the same form as the boundary layer equations for an ideal gas for Prandtl and Lewis numbers equal to unity, Reference 1, Page 182. The integral momentum and energy equations are then written for Prandtl and Lewis numbers equal to unity, Reference 2, Integral Momentum Equation:

$$(1)\quad \frac{d\theta}{dX} = -\left[\left(\frac{\delta^*}{\theta}+2\right)\frac{u_{ex}}{u_e} + \frac{\rho_{ex}}{\rho_e} + \frac{r_x}{r}\right]\theta + c_f/2$$

Integral Energy Equation

$$(2)\quad \frac{d\theta}{dX} = -\left[\frac{u_{ex}}{u_e} + \frac{\rho_{ex}}{\rho_e} + \frac{r_x}{r}\right]\theta - \frac{q_w}{\rho_e u_e H_e^*}$$

with the integral parameters

$$(3)\quad \theta = \delta\int_0^1 \left(1 - \frac{u}{u_e}\right)\frac{\rho u}{\rho_e u_e}\, d\,\frac{y}{\delta}$$

$$(4)\quad \theta = \delta\int_0^1 \left(1 - \frac{H^*}{H_e^*}\right) \frac{\rho u}{\rho_e u_e} d\frac{y}{\delta} \qquad \text{and}$$

$$(5)\quad \frac{\delta^*}{\theta} = \frac{\int_0^1 \left(1 - \frac{\rho u}{\rho_e u_e}\right) d\frac{y}{\delta}}{\int_0^1 \left(1 - \frac{u}{u_e}\right)\frac{\rho u}{\rho_e u_e} d\frac{y}{\delta}}$$

The usual boundary layer coordinate system of x along the body and y normal to the body is employed. It is to be noted that this system of equations does not require explicit knowledge of turbulent shear stresses. The effect of these stresses are included in the skin friction and heat transfer correlations that are required. The procedure is to solve this system of equations (1) - (5) at each station x along the body given the proper empirical skin friction and heat transfer correlations, boundary conditions, and initial conditions.

<u>EQUILIBRIUM FLOW</u> - Profiles for Integral Parameters

First consider the flow in the boundary layer to be in thermodynamic equilibrium. Then the flow properties are determined in the boundary layer from the Mollier diagram, supplied by Ames, given two other flow properties. These two flow properties could be pressure and density or pressure and enthalpy. As seen from the integral parameters θ, φ, and H, the velocity, density, and total enthalpy must be related to coordinate y/δ. The power law profile is assumed.

$$(6)\quad \frac{u}{u_e} = \left(\frac{y}{\delta}\right)^m$$

Now if enthalpy may be connected to velocity, the density may be found from the Mollier diagram since the static pressure is known. The static pressure through the boundary layer is determined from the characteristic solution. The justification for obtaining static pressure through the boundary layer from an inviscid solution will be given in a future progress report.

A modified Crocco equation is found by assuming enthalpy H is a quadratic function of velocity, see Reference 2, or

(7) $H = a + bu + cu^2$. with the boundary conditions

$$\left.\begin{matrix} u = 0 \\ H = H_w \end{matrix}\right) y = 0; \qquad \left.\begin{matrix} u = u_e \\ H = H_e \end{matrix}\right) y = \delta$$

Now for the coefficients in equation (7) become

(8) $a = H_w$

$$b = \frac{H_e - H_w - cu_e^2}{u_e}$$

Then

(9) $$\frac{H^*}{H_e^*} = \frac{H - H_w}{H_e^*} = \frac{H_e^* - cu_e^2}{H_e^*} \quad \frac{u}{u_e} + \frac{cu_e^2}{H_e^*}\left(\frac{u}{u_e}\right)^2$$

Substituting equation (9) in the energy equation and comparing the results with the momentum equation results in

(10) $$\frac{cu_e^2}{H_e^*} = \frac{1}{1 + \frac{C_f^2}{2} \quad (Re/L)\frac{u_e}{u_{ex}}}, \quad Re/L = \frac{\rho_e u_e}{\mu_w}$$

The static enthalpy may be written

(11) $$\frac{h}{h_e} = \frac{h_w}{h_e} + \left[\frac{H_e^*}{h_e} - \frac{cu_e^2}{h_e}\right] \frac{u}{u_e} + (c - 1/2)\frac{u_e^2}{h_e}\left(\frac{u}{u_e}\right)^2$$

Thus, from equations (6) and (11), the flow properties, such as density ρ, temperature T, and atomic mass concentration α, are found from the Mollier diagram. And the integrands in equations (3) - (5) may be evaluated given the power law exponent m.

Skin Friction Correlations

Since turbulent theory has not been developed in a turbulent boundary layer to the extent of relating shear stress directly to temporal mean velocity profiles without empiricism, empirical correlations are necessary for skin friction predications in turbulent boundary layers. The T' skin friction method of Reference 3, generalized for real gas by replacing temperature by enthalpy, is outlined below. The reference enthalpy h' is found

$$(12)\quad h'/h_e = 1 + .035\ M_e^2 + .45\ (h_w/h_e - 1)$$

The reference temperature T' is found from the Mollier diagram given h' + P_e. The viscosity μ' is determined from Reference 4, Table VI given T' and pressure P_e. Also ρ' is determined from the Mollier diagram. The reference skin friction C_f' is computed from the Sivells-Payne correlation, Reference 5.

$$(13)\quad C_f' = \frac{.088\ (\log_{10}R' - 2.3686)}{(\log_{10}R' - 1.5)^3}$$

where the reference Reynolds number R'

$$R' = Re\ \nu_e/\ \nu'\quad,\ \nu = \mu/\rho$$

$$Re = \frac{u_e x}{\nu_e}$$

the compressible skin friction C_f becomes

$$(14)\quad C_f = C_f'\ \rho'/\rho_e$$

All prime quantities are based on the reference temperature T'. The average skin friction C'_F becomes

$$(15)\quad C'_F = \frac{.088}{(\log_{10}R' - 1.5)^2}\quad,\quad C_F = C'_F\ \frac{\rho'}{\rho_e}$$

It is noted that this skin friction method does not depend on pressure gradient. Another skin friction method, developed at Lockheed, is given below. It is termed a modified h' method. The correlation equation is written:

$$(16) \quad \frac{(\theta/\delta)^{m}}{\sqrt{C_f h'/h_e}} = 8.03 + 4.57 \log_{10}\left[R_e \frac{\nu_e}{\nu'} \sqrt{\frac{C_f}{2} \frac{h'}{h_e}} \right]$$

where $\quad R_e = Re\,(\theta/X)$

Equation (16) may be solved for C_f by iteration using the Newton-Raphson method. This method depends on the velocity power law exponent which, in turn, is connected to pressure gradient through the integral equations (1) and (2).

It remains to determine the heat transfer in equation (2). It may be shown that the heat transfer term is directly related to the skin friction similar to Reynolds analogy but retaining the pressure gradient terms. This results in

$$(17) \quad \frac{q_w}{\rho_e u_e H_e^*} = \frac{c u_e^2 / H_e^* - 1}{2} \; C_f \; P_r^{-2/3}$$

Since the initial turbulent boundary layer profile just downstream from transition, depends on various conditions, such as the method of employing transition, it has been decided to input a velocity profile exponent. Then from this power law exponent an equivalent reference Reynolds number is determined from the correlation equation of Sivells and Payne, Reference 5.

$$(18) \quad m_i = \frac{1}{2}\left\{ \frac{1}{1 - 1.47\left[\dfrac{\log R' - 2.3686}{(\log R' - 1.5)^3}\right]^{1/2}} - 1 \right\}$$

Then $Re = R' \dfrac{\nu'}{\nu_e}$. An effective initial distance may be found.

$$(19) \quad x = \frac{Re}{\rho_e u_e / \mu_e}$$

The initial boundary layer thickness δ_i is also input. The local static pressure $P_e(x)$, local density $\rho_e(x)$, body radius $r(x)$, and wall temperature $T_w(x)$ distributions along the body are input. Also total local enthalpy H_e and body length L are input. From the Mollier diagram given pressure P_e and density ρ_e, the enthalpy h_e, temperature T_e, and concentration α_e are found.

The system of equations (1) - (5) may now be solved at each station x along the body given the skin friction equation (14) or (16), the heat transfer equation (17), and the proper boundary conditions and initial conditions. Before the details of solving this system of equations are discussed, consider a boundary layer with weak or zero pressure gradients. If $cu_e^2/H_e^* << 1$, then from equation (9)

$$\frac{H^*}{H_e^*} \cong \frac{u}{u_e}$$

and from equations (3) and (4)

$$\theta \cong \phi$$

This means, in essence, that the momentum equation (1) and the energy equation (2) are similar. For this particular case of weak pressure gradients the approach is to choose the power law exponent m from equation (18) given R' along the body. Then the momentum equation (1) is solved from Adams method for the momentum thickness θ, at various stations along the body. Thus, given the power law exponent m and momentum thickness θ all of the other boundary layer parameters are known.

If cu_e^2/H_e^* is not small, then the following procedure is employed. Given m_i and δ_i at station x_i, the problem is to determine m_{i+1} and δ_{i+1} at station x_{i+1} such that the system of equations (1) - (5) are satisfied. This is accomplished by choosing $m + \delta$ and solving the integral equations (3) - (5) for θ, φ, and H. Then the differential equations (1) and (2) are solved for θ and φ by Adams method using θ, φ, and H, from the integral equations on the right side, and the θ and φ from the differential equations are compared with the θ and φ from the integral equations. The values of m and δ are perturbed through an optimization program until the above θ's and φ's agree within the desired accuracy. The accepted accuracy for matching the θ's and φ's is taken to equal 0.1 per cent and 1 per cent respectively. The desired quantities are printed out and the procedure is repeated for the next increment in x until x = L is reached.

It is interesting to note that using H' skin friction method, a simpler procedure may be utilized. The skin friction C_f, equation (14), pressure gradient parameter, equation (10), and heat transfer parameter, equation (17) may be found at station x_{i+1}. These values do not depend on m_{i+1}, or δ_{i+1} for the h' skin friction method. Then the differential equation (2) may be solved for enthalpy thickness φ_{i+1} given φ_i. The power law exponent m is chosen, and the integral

$$\int_0^1 \left(1 - \frac{H}{H_e^*}\right) \frac{\rho u}{\rho_e u_e} \, d \frac{y}{\delta}$$

is evaluated from Simpson's rule. Then the boundary layer thickness δ_{i+1} is computed from equation (4). Equation (1) is next solved for θ_{i+1} and the error

$$D = \frac{\theta_{Integ} - \theta_{D.E.}}{\theta_{Integ.}}$$

is determined. A new m is chosen from the Newton-Raphson method

$$m_n = m_o - \frac{D}{\Delta D/\Delta m}$$

and the above procedure is repeated until the error $D \leq .001$. Then the required parameters are output, and the above procedure is repeated for the remaining increments in x until $x = L$ is reached.

A simplified block diagram for the turbulent boundary layer routines is given in Figure 1 . This program is presently being checked out. The turbulent boundary layer program with frozen flow will be presented in a future progress report.

NOMENCLATURE

a	- constant in equation (7)
b	- constant in equation (7)
C_f	- local skin friction
C_F	- average skin friction
C	- see equation (10)
H	- total enthalpy
H^*	- $H - H_w$
h	- static enthalpy
L	- reference length
M	- Mach number
m	- power law exponent, equation (6)
P	- Pressure
Pr	- Prandtl number
q_w	- heat transfer rate to the wall
r	- body radius
Re	- Reynolds number based on x
R_θ	- Reynolds number based on θ.
T	- absolute temperature
u	- velocity component along x
v	- velocity component along y
x	- distance along surface of body
y	- distance normal to surface of body
δ	- boundary layer thickness
δ^*	- displacement thickness

Θ	-	momentum thickness
μ	-	viscosity coefficient
ν	-	kinematic viscosity, /
ρ	-	density
Φ	-	energy thickness

Subscripts

e	-	external or edge of boundary layer
w	-	wall conditions
x, y	-	derivative with respect to x, y.

Superscripts

'	-	conditions evaluated at reference temperature T'

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3. Sommer, S. C. and Short, B. J., "Free-Flight Measurements of Turbulent Boundary Layer Skin Friction in the Presence of Severe Aerodynamic Heating at Mach Numbers from 2.8 to 7.0," NACA TN3391, 1955.
4. Hanson, C. F., "Approximations for the Thermodynamic and Transport Properties of High-Temperature Air," NASA TR R-50, 1959.
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BLOCK DIAGRAM OF TURBULENT BOUNDARY LAYER COMPUTER PROGRAM

FIG. 1